GAeL XXX: Junior Talks

University of Warwick - Mathematics Institute

03 - 07 July 2023

Monday July 3rd

10:50am - 11:10am: Art Waeterschoot (KU Leuven)

Title: Resolving wild quotient singularities with differents

Abstract: Given a nice curve C over a p-adic field, ideally one would be able to tell whether it has potentially good reduction from geometric information– for instance its minimal regular model. This is already complicated for elliptic curves, especially if $p = 2$. A decade ago, Lorenzini laid down a framework for studying these questions via resolving wild quotient singularities of arithmetic surfaces. Recently the case of potential good ordinary reduction was completed by Obus and Wewers, using deformation theory and explicit valuation theory – I'll explain another method using Temkin's nonarchimedean analytic differents on metric graphs.

4:00pm - 4:30pm: Ignatius Ngwongwo (University of Ibadan)

Title: Projective elimination ideal of $\overline{M}_{0,n}$

Abstract: Many objects in algebraic geometry vary in algebraically defined families. Moduli theory studies the geometry of families as projective varieties. The problem of finding the right equations that defines a projective variety remains central in algebraic geometry. Often times a set of equations vanishing on the variety in question might be bigger or smaller and thus does not give the right geometry of the variety in the target space. In this talk, I will focus on the moduli spaces $\overline{M}_{0,n}$ (for $n > 2$) of stable genus 0 curves with n-marked points. Keel and Kevelev showed through coordinate free approach that the Segre quadrics generate the homogeneous ideal that defines Rana described the equations of $\overline{M}_{0,n}$ as a projective variety. Lenoid and $\overline{M}_{0,n}$ in $\mathbb{P}^1 \times \mathbb{P}^2 \times \cdots \times \mathbb{P}^{n-3}$ as the set of 2×2 minors of rank one matrices whose entries are cross ratios. My goal will be to describe the equations of $\overline{M}_{0,n}$ in $\mathbb{P}^1 \times \mathbb{P}^2 \times \cdots \times \mathbb{P}^{n-3}$. I will start by describing $\overline{M}_{0,n}$ as a graph of a parametrization in terms of cross ratios and then apply the projective elimination theory to obtain the equations of $\overline{M}_{0,n}$. Afterwords, I will describe the Hilbert function and the corresponding Hilbert series associated to the multihomogeneous coordinate ring of $\mathbb{P}^1 \times \mathbb{P}^2 \times \cdots \times \mathbb{P}^{n-3}$.

Tuesday July 4th

10:50am - 11:20am: Mahrud Sayrafi (University of Minnesota)

Short resolutions of the diagonal and a Horrocks-type splitting criterion in Picard rank 2

Abstract: In 1964, Horrocks proved that a vector bundle on a projective space splits as a sum of line bundles if and only if it has no intermediate cohomology. Motivated by the study of indecomposable vector bundles, in 1978 Beilinson constructed a resolution of the diagonal on $\mathbb{P}^n$ which has been used to

great effect in algebraic geometry. We obtain a Horrocks-type splitting criterion (under an additional hypothesis) for vector bundles over a smooth projective toric variety X of Picard rank 2 using a linear resolution of the diagonal consisting of finite direct sums of line bundles. Since this resolution has length $\dim(X)$, we also prove a new case of a conjecture of Berkesch–Erman–Smith that predicts a version of Hilbert's Syzygy Theorem for virtual resolutions. This is joint work with Michael Brown.

4:00pm - 4:30pm: Dario Faro (University of Pavia)

Title: Gauss-Prym maps on Enriques surface

Abstract: Let C be a complex projective algebraic curve and $\mathcal{L}$ a line bundle on C. One can associate to $\mathcal{L}$ some natural maps between spaces of global sections of certain sheaves on C. These are called Gaussian-Wahl maps and indicated by $\gamma^i_{\mathcal{L}}$, $i \geq 0$. These maps have often been studied to give obstructions for a curve to lie, as a hyperplane section, on a polarized surface (S, H). The most famous example is when (S, H) is a K3 surface. Then $C \in |H|$ is a canonical curve (i.e. $\mathcal{O}_C(H) = \omega_C$) and it has been proved that $\gamma^1_{\omega_C}$ is never surjective. This result becomes very interesting when compared to the fact that for a general element C in the moduli space of curves (of genus g) M_g, $\gamma^1_{\omega_C}$ is surjective for $g \geq 10, g \neq 11$. We consider an analogous problem when (S, H) is a polarized Enriques surface. If $C \in |H|$, then C is a Prym-canonical curve (meaning that $\mathcal{O}_C(H) = \omega_C \otimes \alpha$ where α is a 2-torsion element). We are motivated by the fact that for the general element: (C, α) in the moduli space R_g the Gauss-Prym map $\gamma^1_{\omega_C \otimes \alpha}$ is surjective. We prove that if C is a hyperplane section of an Enriques surface then the Gauss-Prym maps $\gamma^i_{\omega_C \otimes \alpha}$ are surjective for (sufficiently positive i) polarized Enriques surfaces. We conclude that these particular Gauss-Wahl maps don't give obstructions for a curve to lie on an Enriques surface.

Wednesday July 5th

10:50am - 11:20am: Massimiliano Alessandro (University of Genoa and University of Bayreuth (joint PhD))

Title: CHPP surfaces: a new construction method for surfaces of general type with $p_g = q = 2$

Abstract: The classification of surfaces of general type is a classical and long-standing research topic. It turns out that even constructing those with small invariants is a very challenging task. In this talk we construct a family of surfaces of general type having $p_g = q = 2$, $K^2 = 5$ and Albanese map of degree 3. We call it the family of *"CHPP surface"* since it contains the family constructed by Jungkai Alfred Chen and Christopher Derek Hacon in 2006, and coincides with the one considered by Matteo Penegini and Francesco Polizzi in 2013. Our construction yields explicit and global equations and provides the first instance of a new construction method developed in a joint work with Fabrizio Catanese.

Thursday July 6th

10:50am - 11:20am: Girtrude Hamm (University of Nottingham)

Title: Classifying Spherical Gorenstein Fano Four-folds

Abstract: Spherical varieties are a generalisation of toric varieties where instead of a torus we have a spherical homogeneous space G/H. In the same way that lattice polytopes can be associated to toric varieties we can associate lattice polytopes along with some extra data to spherical varieties. In this way the classification of spherical Gorenstein Fano four-folds is reduced to a classification of some lattice polytopes.

2:30pm - 3:00pm: Thibault Poiret (University of Cambridge)

Title: Integral saturation and competing fibre products

Abstract: The fibre product of toric varieties in the category of toric varieties may be smaller than the one in the category of schemes. There is a similar - and related - phenomenon in logarithmic geometry. Log geometry deals with schemes equipped with a well-behaved sheaf of monoids, which we may think of as "the monomial functions". To describe fibre products in log schemes, we can't just take the scheme-theoretic fibre product and equip it with the pushout of the monomial sheaves, because this pushout may not be well-behaved: we have to "make it nice". This process is called integral saturation. We will talk about what integral saturation is, how to compute it, and why we care.

3:05pm - 3:35pm: Johannes Krah (Bielefeld University)

Title: Phantoms and exceptional collections on rational surfaces

Abstract: A smooth projective rational surface over an algebraically closed field admits a full exceptional collection. Building on work of Hille–Perling, Perling, and Vial, we study mutations of (numerically) exceptional collections by analyzing the lattice theoretic behavior of the numerical Grothendieck group. On the one hand, we show that some results, known for del Pezzo surfaces, can be extended to the blow-up of the projective plane in 9 points in very general position. On the other hand, on the blow-up of 10 points in general position we construct an exceptional collection of maximal length which is not full. This disproves a conjecture of Kuznetsov and a conjecture of Orlov.

The talk is based on arXiv:2211.07724 and arXiv:2304.01269.

Friday July 7th

10:50am - 11:20am: Flora Poon (University of Bath)

Title: Kuga-Satake varieties of families of K3 surfaces of Picard rank 14

Abstract: In the 60s, Kuga and Satake constructed a weight one Hodge structure from the second cohomology space of a K3 surface, which gives us a polarised abelian variety called the Kuga-Satake variety associated to the K3 surface. Recently in a paper by Clingher and Malmendier, a few families of K3 surfaces of Picard rank 14 with significance in String theory were examined. The aim of the project is to study the "Kuga-Satake map" from these families of K3 surfaces to the corresponding families of Kuga-Satake varieties as moduli space of PEL type. Another motivation for studying the Kuga-Satake map for families of K3 surfaces of Picard rank 14 is that the correspondence demonstrates, as a particular case, how the type II locally symmetric domains are embedded into the type IV ones.

2:30pm - 3:00pm: Courtney George (University of Kentucky)

Title: Projectivized Toric Vector Bundles as Mori Dream Spaces

Abstract: Mori dream spaces have nice properties and behaviors making them desirable spaces to know and have. In 2010, it was asked: "Under what conditions is a projectivized toric vector bundle a Mori dream space?". Since then, there have been some positive and negative results, but no complete classification yet exists. My result gives a sufficient condition for a rank-r projectivized toric vector bundle to be a Mori dream space, which I hope to continually improve upon to get closer to a necessary condition.

3:05pm - 3:35pm: Oscar Finegan (Cardiff University)

Title: Derived Intersection Products

Abstract: The structure sheaf for an intersection of subschemes is given by taking the tensor product of the structure sheaves of the subschemes that you are intersecting. Derived algebraic geometry (more or less) says that if we replace all of our sheaves by complexes of sheaves, the derived structure complex of the intersection will be the derived tensor product of the original structure complexes. Morally speaking, this new complex "should" encode geometric information about the intersection, namely how badly and in which ways the intersection fails to be transverse, and the question is "What does this complex explicitly look like?" In this talk I will discuss some setup and existing results in this area, as well as my progress on the problem.

4:00pm - 4:30pm: Elena Sammarco (Università degli studi Roma Tre)

Title: Construction of a divisor in the moduli space of cubic fourfolds

Abstract: One of the main open problems of Birational Geometry is to determine the rationality of the cubic hypersurface in $\mathbb{P}^5$, the so-called cubic fourfold: to date there are few explicit results and some conjectures, but no universal criteria. After an overview of the state of the art of the problem on the complex field $\mathbb{C}$, we present a new geometric construction of a divisor in the moduli space of cubic fourfolds which could represent a significant contribution in this general discussion.